Unit 3-33: Mathematics

Rationale

This unit will extend candidates' understanding of mathematics and provide a greater awareness of the analysis of engineering problems and potential solutions.

The unit will also provide a fundamental knowledge of math ematics for candidates who may wish to progress to higher education in related engineering disciplines.

Outcomes

There are four outcomes to this unit. The candidate will be able to:

1. Apply algebraic methods to the solution of problems.

2. Use graphical techniques.

3. Use trigonometric functions and rules to solve problems.

4. Apply differential and integral calculus to the solution of problems.

Connections with other awards

As this unit relates to fundamental mathematics it will therefore provide relevant underpinning knowledge to a wide range of science, engineering and process awards.

Assessment

The outcomes for this unit will be assessed using evidence from a written test. The underpinning knowledge requirements are listed for each outcome. A paper c onsisting of short answer and structured questions based on the test specification will assess these outcomes.

Outcome 1: Apply algebraic methods to the solution of problems

Underpinning knowledge

The candidate will be able to:

1. State the three basic rules of indices and apply them for:

   (a) positive indices
   (b) negative indices
   (c) fractional indices.

2. Evaluate expressions that combine positive, negative and fractional indices.

3. Demonstrate that for all values of a:

   (a) $a^0 = 1$
   (b) $a^{-n} = \dfrac{1}{a^n}$

4. Define a logarithm to any base.

5. Define the inverse of $a^x = y$ as $x = \log_a y$.

6. State the three laws of logarithms:

   (a) $\log(ab) = \log a + \log b$
   (b) $\log\left(\dfrac{a}{b}\right) = \log a - \log b$
   (c) $\log a^n = n \log a$

7. Demonstrate that:

   (a) $\log_b 1 = 0$
   (b) $\log_b b = 1$
   (c) $\log_b x \to -\infty$ as $x \to 0$

8. Apply the above laws to the simplification of logarithmic expressions.

9. Apply the above laws to solve indicial equations.

10. Use the formulae for calculating the areas of:

    (a) rectangles
    (b) parallelograms
    (c) triangles

(d) circles
(e) shapes that are combinations of (a) to (d).

11. Use the formulae for calculating the volumes and surface areas of:

    (a) cylinders
    (b) cuboids
    (c) spheres
    (d) solids that are combinations of (a) to (c).

12. Evaluate formulae.

13. Transform formulae involving brackets.

14. Apply the concept of 'implied' brackets.

15. Transform formulae involving:

    (a) indices
    (b) logarithms.

16. State the inverse notation for trigonometric functions.

17. Transform formulae involving trigonometric functions.

18. Solve simple linear equations.

19. Solve linear simultaneous equations in two and three unknowns:

    (a) by substitution
    (b) by elimination.

20. Solve problems involving ratio.

21. Solve problems involving direct, inverse and combined proportionality.

22. Factorise quadratic expressions (tri nomials).

23. Solve quadratic equations with real roots by:

    (a) factorisation
    (b) use of the quadratic formula.

24. Form and solve quadratic equations that are mathematical models of practical relationships.

25. Sketch labelled graphs of the functions:

    (a) $y = e^{x}$
    (b) $y = e^{-x}$

26. State that natural logarithms are logarithms to the base e.

27. Solve for x, equations of the form $y = Ae^{bx}$.

Outcome 2: Use graphical techniques

Underpinning knowledge

The candidate will be able to:

1. Plot linear graphs from given equations of the form $y = mx + c$.

2. Relate the constants:

   (a) m to the gradient
   (b) c to the graphical intercept.

3. Plot coordinates from a set of given 'experimental' data.

4. Determine the law of a graph using the coordinates of two points on the line.

5. Solve a pair of linear simultaneous equations graphically.

6. State that any graphical solution requires two intersecting graphs and that the technique can be applied to solving a single equation.

7. Apply the graphical technique to solve simple trigonometric equations.

8. Determine the roots of a quadratic equation using the intersection of a parabola with:

   (a) the x axis
   (b) a linear graph.

Outcome 3: Use trigonometric functions and rules to solve problems

Underpinning knowledge

The candidate will be able to:

1. Identify and sketch the graphs over one complete cycle:

   (a) $y = \sin\theta$
   (b) $y = \cos\theta$
   (c) $y = \tan\theta$

2. Obtain the three function values for angles of any magnitude using quadrant rules and a calculator.

3. Solve right-angled triangle problems using trigonometric ratios and/or Pythagoras' theorem.

4. State and use the two rules for a labelled triangle:

   (a) the sine rule
   (b) the cosine rule.

5. Apply the sine rule and/or the cosine rule to the solution of triangle problems.

Outcome 4: Apply differential and integral calculus to the solution of problems

Underpinning knowledge

The candidate will be able to:

1. Relate differentiation to rate of change and to the gradient of the tangent to a curve.

2. Differentiate simple algebraic functions of the form $y = ax^n$ by direct rule, where n is:

   (a) positive
   (b) negative
   (c) fractional.

3. Differentiate algebraic expressions of the form $y = ax^n + bx^{n-1} + \ldots\ldots\ldots\ldots..$

4. Relate integration to the limit of a series and to the area under a g raph.

5. Integrate simple algebraic functions of the form $y = ax^n$ by direct rule, where n is:

   (a) positive
   (b) negative $(n \neq -1)$
   (c) fractional.

6. Evaluate definite integrals of algebraic expressions of the form $y = ax^n + bx^{n-1} + \ldots\ldots\ldots$

7. Use integration to evaluate the area bounded by a curve, the x-axis and two given ordinates.